1. (15 points) a) Let \( A = [v_1 v_2 v_3 v_4] \), where \( v_1 = (1, 0, 1)^T, v_2 = (3, 0, 3)^T, v_3 = (0, 1, 1)^T, v_4 = (2, 4, 6)^T, \) and \( b = (1, 6, 7)^T \). Find the general solution of \( Ax = b \).

b) Are the columns of \( \text{Col}(A) \) linearly independent?

c) Is the system \( Ax = b \) solvable for each \( b \) in \( \mathbb{R}^3 \)?

2. (15 points) a) Let \( A = [v_1 v_2 v_3 v_4] \), where \( v_1 = (2, 3)^T, v_2 = (0, 4)^T, v_3 = (-3, 2)^T, v_4 = (1, 2)^T \). Find bases of \( \text{Nul}(A), \text{Col}(A) \) and \( \text{Row}(A) \).

b) What is the rank of \( A \)? Is the \( \text{Nul}(A) \) orthogonal to \( \text{Col}(A) \)? Explain.

3. (15 points) Let \( u = (2, 1, 0)^T, v = (1, 1, 1)^T \) and \( V = \text{span}\{v\} \).

a) Compute \( w = \text{proj}_V u \).

b) Write \( u \) as the sum of \( w \) and a vector orthogonal to \( v \).

c) Find the distance from \( u \) to \( V \).

4. (20 points) Let \( A = [v_1 v_2 v_3] \), where \( v_1 = (1, 1, 1)^T, v_2 = (1, 1, 1)^T, v_3 = (1, 1, 1)^T \).

a) Find the eigenvalues of \( A \).

b) Find bases of the corresponding eigenspaces.

c) Diagonalize \( A \) (i.e, write it as \( A = PDP^{-1} \)). Do not compute \( P^{-1} \).

d) Using part c), compute \( \text{det}(A) \). Is \( A \) invertible?

5. (20 points) Let \( A = [v_1 v_2 v_3] \), where \( v_1 = (1, 1, 0)^T, v_2 = (1, 0, 1)^T, v_3 = (1, 0, 0)^T \).

a) Use the Gram-Schmidt method to find an orthogonal basis for \( V = \text{Col}(A) \).

b) Find the QR factorization of \( A \).

c) Is \( A \) invertible? Is the system \( Ax = b \) solvable for each \( b \) in \( \mathbb{R}^3 \)? Give the formula for its solution(s) when solvable? Justify your answer.

6. (15 points) a) Write down of the matrix \( A \) corresponding to the quadratic form \( Q(x) = x_1^2 + 2x_1x_2 + 2x_1x_3 \).

b) Is \( Q \) positive definite, negative definite, or indefinite?

c) Orthogonally diagonalize \( A \) and find \( 2A^5 \) and \( \text{det}(2A^5) \).