1) (20 points) Let \( A = [v_1 v_2 v_3] \) with 
\( v_1 = (2, -1, 1)^T, \ v_2 = (0, 8, -2)^T \) and 
\( v_3 = (6, 5, 1)^T \) and \( b = (10, 3, 3)^T \).

a) Are the columns of \( A \) linearly independent? Is \( A \) invertible?
b) Find the general solution of \( Ax = b \).
c) What is a basis and the dimension of \( \text{Col}(A) \)? Is \( b \) in \( \text{Col}(A) \)?
d) What is the rank of \( A \) and the dimension of the null space of \( A \)?

2) (15 points) Let
\( u = (-1, 1, 1)^T, \ v = (3, -1, 2)^T \) and \( V = \text{span}\{v\} \).

a) Find the projection \( w = \text{proj}_V u \) of \( u \) onto \( V \).
b) Write \( u \) as the sum of \( w \) and a vector orthogonal to \( v \).
c) Find the distance from \( u \) to \( V \).

3) (20 points) Let \( A = [x_1 x_2 x_3] \) with 
\( x_1 = (1, -1, -1)^T, \ x_2 = (0, 3, 3)^T \) and 
\( x_3 = (3, 2, 4)^T \).

a) Use the Gram-Schmidt process to find an orthogonal basis for \( V = \text{Col}(A) \).
b) Find the QR factorization of \( A \). Express \( R \) in terms of \( Q \) and \( A \), but don’t compute it.
c) Use part b) to determine if \( A \) is invertible (no computation is required).

4) (20) Let \( A = [v_1 v_2 v_3] \) with 
\( v_1 = (7, -3, 2)^T, \ v_2 = (1, 3, 2)^T \) and 
\( v_3 = (-2, 6, 2)^T \).

a) Find the eigenvalues of \( A \).
b) Find bases for the corresponding eigenspaces.
c) Diagonalize \( A \) (i.e., write it as \( A = PDP^{-1} \)). Do not compute \( P^{-1} \). Use it to find \( \det(A) \).

5) (25 points) a) Write down the matrix \( A \) corresponding to the quadratic form
\( Q(x) = 5x_1^2 + 4x_1x_2 + 2x_2^2 \).

b) Is \( Q \) positive definite, negative definite, or indefinite?
c) Orthogonally diagonalize \( A \) and find \( A^{10} \).
d) Find the change of variable \( x = Py \) that transforms \( Q \) into a quadratic form with just square terms. Compute this new form.