

CALCULUS 211-FINAL EXAM-DECEMBER 17, 2008

1) For the line and the plane given by $x = 3 - t$, $y = t + 1$, $z = 3t$ and $x + y - z = 1$

a) Determine the coordinates of the point of intersection of the line and the plane

b) Determine the cosine of the angle between the line and a line normal to the given plane

2) For the space curve $x = t + 1$, $y = \sqrt{2t + 1}$, $z = (t^2 - 4)$

a) Determine the equation of the line tangent to the curve at $t = 1$

b) Determine the acceleration vector at $t = 1$

3) Find all the local relative and absolute extrema of the function

$f(x, y) = x^2 + 3y - 3xy$ bounded by the region $y = x$, $y = 0$, $x = 2$

4) Using Lagrange multipliers find the maximum and minimum values of the function

$f(x, y) = xe^y$ subject to the constraint $x^2 + y^2 = 2$

5) a) Determine the directional derivative of $f = e^{xy+z}$ at the point $(1, -1, 1)$ in the direction from $(1, 3, 1)$ to $(1, 6, -3)$

b) Determine the equation of the plane tangent to the surface $e^{xy+z} = 1$ at the point $(1, -1, 1)$

6) Evaluate, using cylindrical coordinates

$\iiint e^z dV$ for the region between $z = 0$ and $z = -\sqrt{4 - x^2 - y^2}$ and outside

the cylinder $x^2 + y^2 = 3$

7) Evaluate the line integral $\int_C xz ds$ where C is the straight line segment from $(1, 0, 1)$ to $(2, -2, 2)$

8) For the conservative vector field $\mathbf{F} = 2xy\mathbf{i} + (x^2 - 1)\mathbf{j}$

determine its potential $f(x, y)$ and evaluate the line integral $\int_{(1,0)}^{(3,1)} \mathbf{F} \cdot d\mathbf{r}$.

9) Evaluate directly as a line integral $\oint x dy$ for the closed curve which encloses the region

formed by the curves $y = x^2$ and $y = 4$, oriented counterclockwise

10) Use Green's Theorem for the vector field $\mathbf{F} = (y^3 - \ln x)\mathbf{i} + (\sqrt{y^2 + 1} + 3x)\mathbf{j}$ to evaluate the line integral $\oint \mathbf{F} \cdot d\mathbf{r}$ over the boundary of the region formed by the curves $x = y^2$ and $x = 4$, as a double integral over this region, oriented counterclockwise